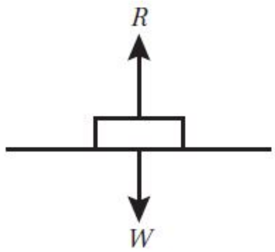
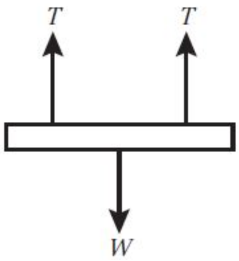
Exercise 4A

1 R is the normal reaction of the table on the box.
W is the weight of the box.

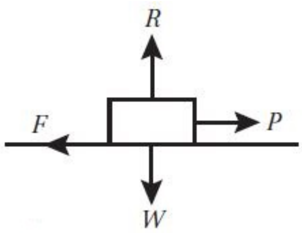


2 T is the tension in each of the ropes.
W is the weight of the bar.



3 W is the weight of the apple.

4 R is the normal reaction of the road on the car.
W is the weight of the car.
F is the sum of the frictional forces on the car.
P is the forward force produced by the car's engine.



5 W is the weight of the rescuer.
T is the tension in the rope.

6 Although its speed is constant, the satellite is continuously changing direction. This means the velocity changes. Therefore, there must be a resultant force acting on the satellite.

7 5 N

8 Since each particle is stationary, the overall force in each case is zero.

a Considering vertical forces:
$P - 10 = 0$
$P = 10$ N

8 b Considering horizontal forces only:
$P - 30 = 0$
$P = 30$ N

c Considering horizontal forces only:
$P + 1.5P - 50 = 0$
$2.5P = 50$
$P = 20$ N

9 a Since the platform is moving at constant velocity, the total vertical force is zero.
$T + T = 400$
$T = 200$
The tension in each rope is 200 N.

b If the tension in each rope is reduced by 50 N, there is a resultant downward force on the platform. It will therefore accelerate downward.

10 Since the particle is at rest, both horizontal and vertical forces must be balanced.
Considering horizontal forces only:
$p - 50 = 0$
$p = 50$
Considering vertical forces only:
$5q - (q + 10) - 3p = 0$
$4q - 10 - (3 \times 50) = 0$
$4q = 160$
$q = 40$
The values of p and q are 50 and 40 respectively.

11 Since the particle is moving with constant velocity, both horizontal and vertical forces must be balanced.
Considering horizontal forces only:
$2P + Q = 25$
$Q = 25 - 2P$
Considering vertical forces only:
$3P - 2Q = 20$
Substituting for Q:
$3P - 2 \times (25 - 2P) = 20$
$3P - 50 + 4P = 20$
$7P = 20 + 50 = 70$
$P = 10$ N
Using this value of P in the horizontal equation:
$(2 \times 10) + Q = 25$
$Q = 25 - 20 = 5$
$Q = 5$ N
P is 10 N and Q is 5 N.

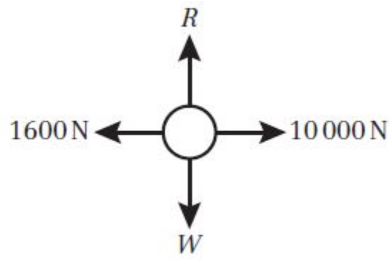
12 a i Overall horizontal force = 100 – 100 = 0
Overall vertical force = 40 – 20 = 20
The resultant force is 20 N upward.

ii The particle accelerates vertically upward.

12 b i Overall horizontal force = 25 – 5 = 20
Overall vertical force = 10 – 10 = 0
The resultant force is 20 N to the right.

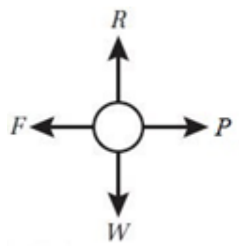
ii The particle accelerates to the right.

13 a R is the normal reaction of the road on the car.
W is the weight of the car.
The forward thrust of the car's engine acts to the right in the diagram.
The car is travelling to the right (positive direction).
The frictional forces on the car are acting to the left.



b Considering horizontal forces only:
Resultant force = 10 000 – 1600
There is no overall vertical force: R and W must be balanced, otherwise the car would lift off the road or sink into it.
The resultant force is 8400 N in the direction of travel.

14 a R is the normal reaction of the road on the car.
W is the weight of the car.
P is the driving force produced by the car's engine.
F is the resistance to the car's motion.



b The magnitude of the driving force is eight times the magnitude of the resistance force, so

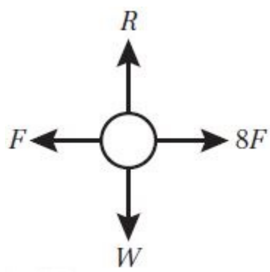
$P = 8F$

The resultant force is the difference between the forward force P and the resistance force F, so

$8F - F = 7F = 4200$

$F = \dfrac{4200}{7} = 600$

The magnitude of the resistance force is 600 N.



Exercise 4B

1 a $(-\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) = (3\mathbf{i} + 2\mathbf{j})$
The resultant force is $(3\mathbf{i} + 2\mathbf{j})$ N.

b $\begin{pmatrix}5\\3\end{pmatrix}+\begin{pmatrix}-3\\-6\end{pmatrix}=\begin{pmatrix}2\\-3\end{pmatrix}$

The resultant force is $\begin{pmatrix}2\\-3\end{pmatrix}$ N.

c $(\mathbf{i} + \mathbf{j}) + (5\mathbf{i} - 3\mathbf{j}) + (-2\mathbf{i} - \mathbf{j}) = (4\mathbf{i} - 3\mathbf{j})$
The resultant force is $(4\mathbf{i} - 3\mathbf{j})$ N.

d $\begin{pmatrix}-1\\4\end{pmatrix}+\begin{pmatrix}6\\0\end{pmatrix}+\begin{pmatrix}-2\\-7\end{pmatrix}=\begin{pmatrix}3\\-3\end{pmatrix}$

The resultant force is $\begin{pmatrix}3\\-3\end{pmatrix}$ N.

2 a $\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3=0$

$\Rightarrow (2\mathbf{i}+7\mathbf{j})+(-3\mathbf{i}+\mathbf{j})+\mathbf{F}_3=0$

$\Rightarrow \mathbf{F}_3=-(2\mathbf{i}+7\mathbf{j})-(-3\mathbf{i}+\mathbf{j})$

$=-2\mathbf{i}-7\mathbf{j}+3\mathbf{i}-\mathbf{j}$

$=\mathbf{i}-8\mathbf{j}$

b $\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3=0$

$\Rightarrow (3\mathbf{i}-4\mathbf{j})+(2\mathbf{i}+3\mathbf{j})+\mathbf{F}_3=0$

$\Rightarrow \mathbf{F}_3=-(3\mathbf{i}-4\mathbf{j})-(2\mathbf{i}+3\mathbf{j})$

$=-3\mathbf{i}+4\mathbf{j}-2\mathbf{i}-3\mathbf{j}$

$=-5\mathbf{i}+\mathbf{j}$

3 Since object is in equilibrium:

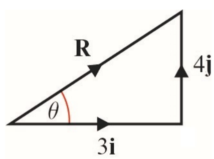
$$\begin{pmatrix}a\\2b\end{pmatrix}+\begin{pmatrix}-2a\\-b\end{pmatrix}+\begin{pmatrix}3\\-4\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{pmatrix}a\\2b\end{pmatrix}+\begin{pmatrix}-2a\\-b\end{pmatrix}=\begin{pmatrix}-3\\4\end{pmatrix}$$

$$\begin{pmatrix}-a\\b\end{pmatrix}=\begin{pmatrix}-3\\4\end{pmatrix}$$

$a = 3$ and $b = 4$

4 a $(3\mathbf{i} + 4\mathbf{j})$



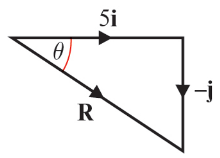
4 a i $\sqrt{3^2+4^2}=\sqrt{25}$

The resultant force is 5 N.

ii $\tan\theta=\frac{4}{3}$

The force makes an angle of 53.1° with **i**.

b (5**i** – **j**)



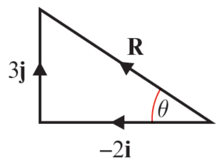
i $\sqrt{5^2+1^2}=\sqrt{26}$

The resultant force is $\sqrt{26}$ N.

ii $\tan\theta=\frac{1}{5}$

The force makes an angle of 11.3° with **i**.

c (−2**i** + 3**j**)



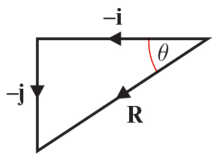
i $\sqrt{2^2+3^2}=\sqrt{13}$

The resultant force is $\sqrt{13}$ N.

ii $\tan\theta=\frac{3}{2}$

$\theta=56.3°$ This is the angle made with the negative **i** vector
Angle made with the positive **i** vector = 180 − θ
The force makes an angle of 123.7° with **i**.

d



i $\sqrt{1^2+1^2}=\sqrt{2}$

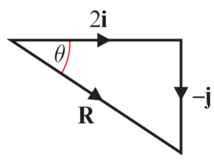
The resultant force is $\sqrt{2}$ N.

4 d ii $\tan\theta = \frac{1}{1}$

$\theta = 45°$. This is the angle made with the negative **i** vector.
The obtuse angle made with the positive **i** vector = $180 - \theta$
The force makes an angle of 135° with **i**.

5 a i $(-2\mathbf{i}+\mathbf{j})+(5\mathbf{i}+2\mathbf{j})+(-\mathbf{i}-4\mathbf{j})=(2\mathbf{i}-\mathbf{j})$
The resultant vector is $(2\mathbf{i}-\mathbf{j})$ N.

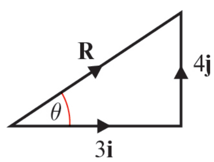
ii $\sqrt{2^2+1^2}=\sqrt{5}$
The magnitude of the resultant vector is $\sqrt{5}$ N.



iii $\tan\theta = \frac{1}{2}$
$\theta = -26.6°$ This is the angle made from **east**, with **anticlockwise** defined as positive.
The **bearing** is the angle made from **north**, with **clockwise** defined as positive = $90 - \theta$
The force acts at a bearing of 116.6°.

b i $(-2\mathbf{i}+\mathbf{j})+(2\mathbf{i}-3\mathbf{j})+(3\mathbf{i}+6\mathbf{j})=(3\mathbf{i}+4\mathbf{j})$
The resultant vector is $(3\mathbf{i}+4\mathbf{j})$ N

ii $\sqrt{3^2+4^2}=\sqrt{25}$
The resultant force is 5 N.



iii $\tan\theta = \frac{4}{3}$
$\theta = 53.1°$ This is the angle made from **east**, with **anticlockwise** defined as positive.
The **bearing** is the angle made from **north**, with **clockwise** defined as positive = $90 - \theta$
The force acts at a bearing of 36.9°.

6 Since the object is in equilibrium:
$(a\mathbf{i} - b\mathbf{j}) + (b\mathbf{i} + a\mathbf{j}) + (-4\mathbf{i} - 2\mathbf{j}) = 0$
Considering **i** components:
$a + b - 4 = 0$
so $b = 4 - a$ **(1)**
Considering **j** components:
$-b + a - 2 = 0$
Substituting $b = 4 - a$ from **(1)**:
$-(4 - a) + a - 2 = 0$
$2a = 2 + 4 = 6$
$a = 3$ **(2)**

Substituting **(2)** into **(1)**:
$b = 4 - 3 = 1$
The values of a and b are 3 and 1, respectively.

7 Since the object is in equilibrium:
$(2a\mathbf{i} + 2b\mathbf{j}) + (-5b\mathbf{i} + 3a\mathbf{j}) + (-11\mathbf{i} - 7\mathbf{j}) = 0$

Considering **i** components:
$2a - 5b - 11 = 0$ **(1)**

Considering **j** components:
$2b + 3a - 7 = 0$ **(2)**

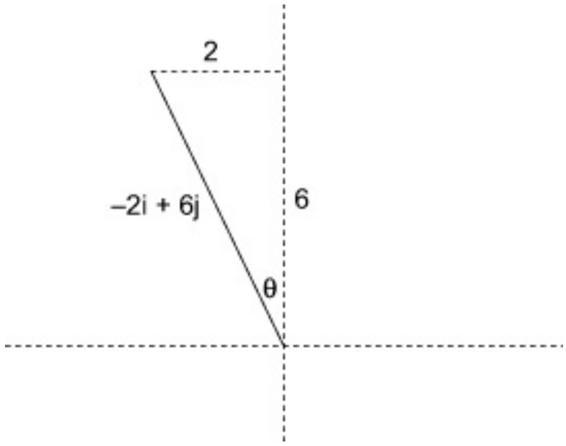
equation **(1)** × 3 → $6a - 15b - 33 = 0$ **(3)**
equation **(2)** × 2 → $6a + 4b - 14 = 0$ **(4)**

Subtracting (**4**) from (**3**):
$-15b - 33 - 4b - (-14) = 0$
$-19b = 33 - 14$
$b = -1$
Substituting this value into equation (**1**):
$2a - 5(-1) - 11 = 0$
$2a = 11 - 5 = 6$
The values of a and b are 3 and -1, respectively.

8 **a** $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$
$\Rightarrow (-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = 0$
$(-3 + 1 + p)\mathbf{i} + (7 - 1 + q)\mathbf{j} = 0$
$p = 2,\ q = -6$

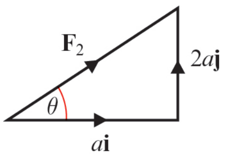
b $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$
$= (-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j})$
$= -2\mathbf{i} + 6\mathbf{j}$
$|\mathbf{R}| = \sqrt{(-2)^2 + 6^2}$
$= \sqrt{4 + 36}$
$= \sqrt{40}$
$= 6.32$ N

8 c



$$\tan\theta = \frac{2}{6}$$
$$\theta = 18^\circ$$

9 a $\mathbf{F}_2 = (a\mathbf{i} + 2a\mathbf{j})$



$$\tan\theta = \frac{2a}{a} = 2$$

$\mathbf{F}_2$ makes an angle of 63.4° with **i**.

b $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (3\mathbf{i} - 2\mathbf{j}) + (a\mathbf{i} + 2a\mathbf{j})$

i vector = $3 + a$ **j** vector = $-2 + 2a$

In the vector ($13\mathbf{i} + 10\mathbf{j}$):
i vector = 13 **j** vector = 10

Let θ_1 = the angle of vector **R** and θ_2 = the angle of vector ($13\mathbf{i} + 10\mathbf{j}$)

Since the vectors are parallel, $\theta_1 = \theta_2$ so $\tan\theta_1 = \tan\theta_2$:

$$\tan\theta_1 = \frac{\mathbf{j}\text{ vector}}{\mathbf{i}\text{ vector}} = \frac{-2+2a}{3+a} \qquad \tan\theta_2 = \frac{\mathbf{j}\text{ vector}}{\mathbf{i}\text{ vector}} = \frac{10}{13}$$

$$\Rightarrow \frac{-2+2a}{3+a} = \frac{10}{13}$$

$(-2 + 2a) \times 13 = (3 + a) \times 10$
$16a = 56$
$a = 3.5$

10 a Since the particle P is in equilibrium:

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$$\begin{pmatrix}-7\\-4\end{pmatrix}+\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

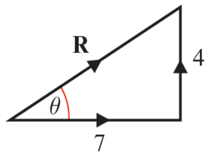
$$\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}7\\4\end{pmatrix}+\begin{pmatrix}-4\\-2\end{pmatrix}$$

$$\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}3\\2\end{pmatrix}$$

The values are $a = 3$, $b = 2$

b $\mathbf{R} = \mathbf{F}_2 + \mathbf{F}_3$

$$\mathbf{R}=\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}7\\4\end{pmatrix}$$



i $|\mathbf{R}| = \sqrt{7^2 + 4^2} = \sqrt{65}$

The magnitude of **R** is $\sqrt{65}$ N.

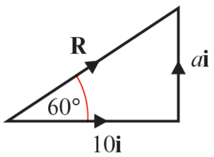
ii $\tan\theta = \dfrac{4}{7}$

$\theta = 29.7\ldots°$

R acts at 30° above the horizontal (to 2 s.f.)

Challenge

Redrawing the diagram as a closed triangle:



$\tan 60 = \dfrac{a}{10}$

$a = 10\tan 60 = 10 \times \sqrt{3}$

$$\mathbf{R}=\begin{pmatrix}10\\a\end{pmatrix}=\begin{pmatrix}10\\10\sqrt{3}\end{pmatrix}$$

$$|\mathbf{R}| = \sqrt{10^2 + \left(10\sqrt{3}\right)^2} = \sqrt{100+300} = \sqrt{400}$$

The value of a is 17.3 (to 3 s.f.), and the magnitude of the resultant force is 20 N.

Exercise 4C

1 $F = ma$

$120 = 400a$

$a = 0.3$

The acceleration is $0.3\,\text{m}\,\text{s}^{-2}$

2 $W = mg$

$= 4 \times 9.8$

$= 39.2$

The weight of the particle is $39.2\,\text{N}$

3 $F = ma$

$30 = 1.2m$

$m = 25$

The mass of the object is $25\,\text{kg}$.

4 On Earth: $W = 735$ N, $g = 9.8$ m s^{-2}, $m = ?$

$W = mg$
$735 = m \times 9.8$
$m = 735 \div 9.8 = 75$ kg

On the moon: $W = 120$ N, $g = ?$, $m = 75$
$W = mg$
$120 = 75 \times g$
$g = 120 \div 75 = 1.6$

On the Moon, the acceleration due to gravity is 1.6 m s^{-2}.

5 Always resolve in the direction of acceleration.

a $R(\uparrow)$, $P - 2g = 2 \times 3$

$P = 25.6$

The magnitude of P is $25.6\,\text{N}$

b $R(\downarrow)$, $4g + 10 - P = 4 \times 2$

$49.2 - P = 8$

$P = 41.2$

The magnitude of P is $41.2\,\text{N}$

6 a $R(\downarrow)$, $mg - 10 = m \times 5$

$9.8m - 10 = 5m$

$m = 2.1 \quad (2\text{ s.f.})$

The mass of the body is 2.1 kg

b $R(\uparrow)$, $20 - mg = m \times 2$

$20 - 9.8m = 2m$

$m = 1.7 \quad (2\text{ s.f.})$

The mass of the body is 1.7 kg

7 a $R(\downarrow)$, $2g - 8 = 2a$

$5.8 = a$

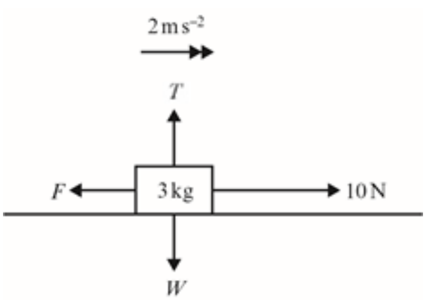
The acceleration of the body is 5.8 m s^{-2}

b $R(\uparrow)$, $100 - 8g = 8a$

$2.7 = a$

The acceleration of the body is 2.7 m s^{-2}

8 W and T can be ignored, as they act at right angles to the motion.



Resultant force = ma
m = 3 kg, a = 2 ms^{-2}
R (→), $10 - F = 3 \times 2 = 6$
$F = 10 - 6$
The force due to friction is 4 N.

9 a $u = 0,\ v = 3,\ s = 5,\ a = ?$

$v^2 = u^2 + 2as$

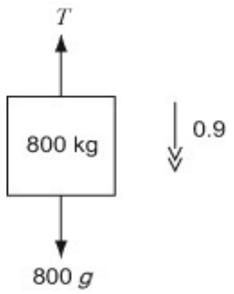
$3^2 = 0^2 + 2a \times 5$

$9 = 10a$

$a = 0.9$

The acceleration of the lift is 0.9 m s^{-2}

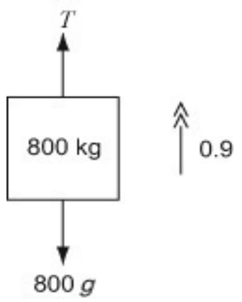
9 b



$$R(\downarrow),\quad 800g - T = 800 \times 0.9$$
$$7840 - T = 720$$
$$T = 7120$$

The tension in the cable is 7120 N.

c



$$R(\uparrow),\quad T - 800g = 800 \times 0.9$$
$$T - 7840 = 720$$
$$T = 8560$$

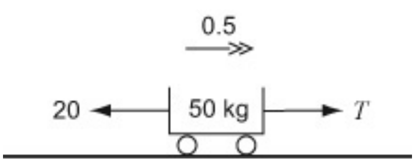
The tension in the cable is 8560 N.

10 a $u = 0,\ v = 1,\ t = 2,\ a = ?$

$$v = u + at$$
$$1 = 0 + a \times 2$$
$$a = 0.5$$

The acceleration of the trolley is $0.5\,\text{m}\,\text{s}^{-2}$

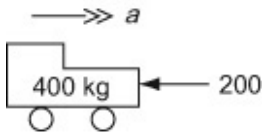
b



$$R(\rightarrow),\quad T - 20 = 50 \times 0.5$$
$$T = 45$$

The tension in the rope is 45 N.

11 a



$R(\rightarrow),\quad -200 = 400a$

$a = -0.5$

$u = 16,\ v = 0,\ a = -0.5,\ t = ?$

$v = u + at \quad (\rightarrow)$

$0 = 16 - 0.5t$

$0.5t = 16$

$t = 32$

It takes 32 s for the van to stop.

b $u = 16,\ v = 0,\ a = -0.5,\ s = ?$

$v^2 = u^2 + 2as \quad (\rightarrow)$

$0^2 = 16^2 + 2(-0.5)s$

$0 = 256 - s$

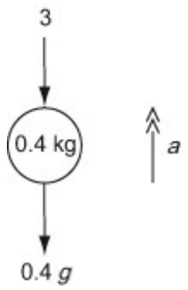
$s = 256$

The van travels 256 m before it stops.

c Air resistance is unlikely to be of constant magnitude. (It is usually a function of speed.)

Challenge

a



$R(\uparrow),\quad -3 - 0.4g = 0.4a$

$a = -17.3$

$u = 10,\ v = 0,\ a = -17.3,\ s = ?$

$v^2 = u^2 + 2as \quad (\uparrow)$

$0 = 10^2 + 2(-17.3)s$

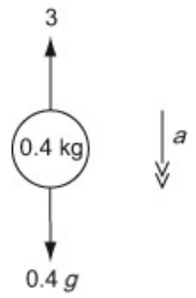
$0 = 100 - 34.6s$

$s = 2.89\ldots = 2.9 \quad (2\text{ s.f.})$

The stone rises to a height of 2.9 m above the bottom of the pond.

Challenge

b



$$R(\downarrow),\quad 0.4g - 3 = 0.4a$$
$$0.92 = 0.4a$$
$$a = 2.3$$

$u = 0,\ s = \frac{100}{34.6},\ a = 2.3,\ v = ?$
$v^2 = u^2 + 2as \quad (\downarrow)$
$v^2 = 0^2 + 2 \times 2.3 \times \frac{100}{34.6}$
$v = 3.646.. = 3.6 \quad (2 \text{ s.f.})$

The stone hits the bottom of the pond with speed $3.6\,\text{m}\,\text{s}^{-1}$

c $u = 10,\ v = 0,\ a = -17.3,\ t = ?$
$v = u + at \quad (\uparrow)$
$0 = 10 - 17.3t,$
$t_1 = \frac{10}{17.3} = 0.57803\ldots$

$u = 0,\ a = 2.3,\ s = \frac{100}{34.6},\ t = ?$
$s = ut + \frac{1}{2}at^2 \quad (\downarrow)$
$\frac{100}{34.6} = 0 + \frac{1}{2} \times 2.3t_2^2$
$t_2^2 = \dfrac{2 \times 100}{2.3 \times 34.6} = 2.51319$
$t_2 = 1.585$

$t_1 + t_2 = 0.57803 + 1.585 = 2.16$

The total time is 2.16 s (3 s.f.)

Exercise 4D

1 a $F = (\mathbf{i} + 4\mathbf{j})$, $m = 2$, $\mathbf{a} = ?$

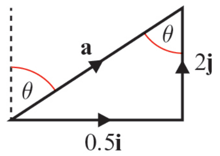
$F = m\mathbf{a}$

$(\mathbf{i} + 4\mathbf{j}) = 2\mathbf{a}$

$\mathbf{a} = \dfrac{(\mathbf{i}+4\mathbf{j})}{2}$

The acceleration of the particle is $(0.5\mathbf{i} + 2\mathbf{j})$ m s^{-2}.

b



$|\mathbf{a}| = \sqrt{0.5^2 + 2^2} = \sqrt{4.25}$

The magnitude of the acceleration is 2.06 m s^{-2}.

Using Z angles (see diagram), bearing = θ

$\tan\theta = \dfrac{0.5}{2}$

$\theta = 14°$

The bearing of the acceleration is 014°.

2 $F = (4\mathbf{i} + 3\mathbf{j})$, $\mathbf{a} = (20\mathbf{i} + 15\mathbf{j})$, $m = ?$

$F = m\mathbf{a}$

$(4\mathbf{i} + 3\mathbf{j}) = m \times (20\mathbf{i} + 15\mathbf{j})$

$m = \dfrac{(4\mathbf{i}+3\mathbf{j})}{(20\mathbf{i}+15\mathbf{j})} = \dfrac{1}{5}$

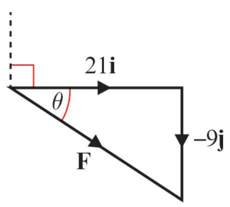
The mass of the particle is 0.2 kg.

3 a $\mathbf{a} = (7\mathbf{i} - 3\mathbf{j})$, $m = 3$, $F = \mathbf{?}$

$F = m\mathbf{a}$

$= 3 \times (7\mathbf{i} - 3\mathbf{j})$

$= (21\mathbf{i} - 9\mathbf{j})$



b

$|\mathbf{F}| = \sqrt{21^2 + 9^2} = \sqrt{522}$

The force has a magnitude of 22.8 N (3 s.f.)

$\tan\theta = \dfrac{9}{21}$

$\theta = 23.19\ldots°$

But bearing = $90° + \theta$ (see diagram)

The force acts at a bearing of 113° (to the nearest degree).

4 a $\mathbf{F}_1 = (2\mathbf{i} + 7\mathbf{j})$, $\mathbf{F}_2 = (-3\mathbf{i} + \mathbf{j})$, $m = 0.25$

$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$

$(2\mathbf{i} + 7\mathbf{j}) + (-3\mathbf{i} + \mathbf{j}) = 0.25\mathbf{a}$

$(-\mathbf{i} + 8\mathbf{j}) = 0.25\mathbf{a}$

$\mathbf{a} = \dfrac{(-\mathbf{i}+8\mathbf{j})}{0.25}$

The acceleration is $(-4\mathbf{i} + 32\mathbf{j})$ m s^{-2}.

b $\mathbf{F}_1 = (3\mathbf{i} - 4\mathbf{j})$, $\mathbf{F}_2 = (2\mathbf{i} + 3\mathbf{j})$, $m = 6$

$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$

$(3\mathbf{i} - 4\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) = 6\mathbf{a}$

$(5\mathbf{i} - \mathbf{j}) = 6\mathbf{a}$

$\mathbf{a} = \dfrac{(5\mathbf{i}-\mathbf{j})}{6}$

The acceleration is $\left(\dfrac{5}{6}\mathbf{i} - \dfrac{1}{6}\mathbf{j}\right)$ m s^{-2}.

c $\mathbf{F}_1 = (-40\mathbf{i} - 20\mathbf{j})$, $\mathbf{F}_2 = (25\mathbf{i} + 10\mathbf{j})$, $m = 15$

$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$

$(-40\mathbf{i} - 20\mathbf{j}) + (25\mathbf{i} + 10\mathbf{j}) = 15\mathbf{a}$

$(-15\mathbf{i} - 10\mathbf{j}) = 15\mathbf{a}$

$\mathbf{a} = \dfrac{(-15\mathbf{i}-10\mathbf{j})}{15}$

The acceleration is $\left(-\mathbf{i} - \dfrac{2}{3}\mathbf{j}\right)$ m s^{-2}.

d $\mathbf{F}_1 = 4\mathbf{j}$, $\mathbf{F}_2 = (-2\mathbf{i} + 5\mathbf{j})$, $m = 1.5$

$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$

$4\mathbf{j} + (-2\mathbf{i} + 5\mathbf{j}) = 1.5\mathbf{a}$

$(-2\mathbf{i} + 9\mathbf{j}) = 1.5\mathbf{a}$

$\mathbf{a} = \dfrac{(-2\mathbf{i}+9\mathbf{j})}{1.5}$

The acceleration is $\left(-\dfrac{4}{3}\mathbf{i} + 6\mathbf{j}\right)$ m s^{-2}.

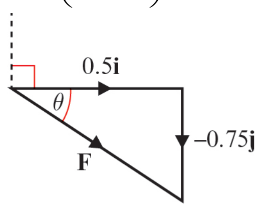
5 a Resultant force, $F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$$F = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -5 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

$F = m\mathbf{a}$

$$8\mathbf{a} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

$$\mathbf{a} = \begin{pmatrix} 0.5 \\ -0.75 \end{pmatrix}$$



5 a $|\mathbf{a}| = \sqrt{0.5^2 + 0.75^2} = \sqrt{0.8125}$

$\tan\theta = \dfrac{0.75}{0.5}$

$\theta = 56°$

But bearing = 90° + θ (see diagram)

The acceleration has a magnitude of 0.901 m s^{-2} and acts at a bearing of 146°.

b $s = 20,\ u = 0,\ a = 0.901$

$s = ut + \frac{1}{2}at^2$

$20 = (0 \times t) + \left(\frac{1}{2} \times 0.901 \times t^2\right)$

$t^2 = \dfrac{20 \times 2}{0.901} = 44.39$

The particle takes 6.66 s to travel 20 m.

6 $\mathbf{R} = (2\mathbf{i} + 3\mathbf{j}) + (p\mathbf{i} + q\mathbf{j})$

Since $\mathbf{R}$ is parallel to $(-\mathbf{i} + 4\mathbf{j})$,

$\mathbf{R} = (-k\mathbf{i} + 4k\mathbf{j})$ where k is a constant

$(2\mathbf{i} + 3\mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = (-k\mathbf{i} + 4k\mathbf{j})$

Collecting **i** terms: $2 + p = -k$

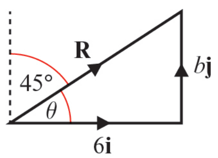
so $k = -2 - p$

Collecting **j** terms: $3 + q = 4k$

Substituting for k: $3 + q = 4(-2 - p)$

so $3 + q = -8 - 4p$

$4p + q + 11 = 0$



7 a $\theta = 90° - 45°$ (see diagram)

$\tan 45° = \dfrac{b}{6}$

$b = 6 \times \tan 45° = 6 \times 1$

The value of b is 6.

b $|\mathbf{R}| = \sqrt{6^2 + 6^2} = \sqrt{72}$

The magnitude of $\mathbf{R}$ is $6\sqrt{2}$ N (8.49 N to 3.s.f)

c $F = 6\sqrt{2},\ m = 4,\ a = ?$

$F = ma$

$6\sqrt{2} = 4a$

The magnitude of the acceleration of the particle is $\dfrac{3\sqrt{2}}{2}$ m s^{-2} (2.12 m s^{-2} to 3 s.f.)

7 d $t = 5$, $u = 0$, $a = \dfrac{3\sqrt{2}}{2}$, $s = ?$

$$s = ut + \frac{1}{2}at^2$$

$$s = (0 \times 5) + \left(\frac{1}{2} \times \frac{3\sqrt{2}}{2} \times 5^2\right)$$

$$s = \frac{75\sqrt{2}}{4}$$

In the first 5 s the particle travels $\dfrac{75\sqrt{2}}{4}$ m (26.5 m to 3 s.f.).

8 a Since particle is in equilibrium, $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$(-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = 0$
Collecting **i** terms: $-3 + 1 + p = 0$
Collecting **j** terms: $7 - 1 + q = 0$

The value of p is 2, and the value of q is -6.

b When $\mathbf{F}_2$ is removed, resultant force, $F = \mathbf{F}_1 + \mathbf{F}_3$
$F = (-3\mathbf{i} + 7\mathbf{j}) + (2\mathbf{i} - 6\mathbf{j}) = (-\mathbf{i} + \mathbf{j})$
The magnitude of this force is $\sqrt{1^2 + 1^2} = \sqrt{2}$

$s = 12$, $t = 10$, $u = 0$, $a = ?$

$$s = ut + \frac{1}{2}at^2$$

$$12 = (0 \times 20) + \left(\frac{1}{2} \times a \times 10^2\right)$$

$$12 = 50a$$

$$a = \frac{12}{50} = \frac{6}{25}$$

$$F = \sqrt{2},\ a = \frac{6}{25}$$

$$F = ma$$

$$\sqrt{2} = m \times \frac{6}{25}$$

The mass of the particle is $\dfrac{25\sqrt{2}}{6}$ kg.

9 Resultant force, $F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$F = (5\mathbf{i} + 6\mathbf{j}) + (2\mathbf{i} - 2\mathbf{j}) + (-\mathbf{i} - 4\mathbf{j}) = 6\mathbf{i}$
Since this has only a single component, the magnitude of the force is 6 N.
$a = 7$
$F = ma$
$6 = m \times 7$
$m = 6 \div 7$
The mass of the particle is 0.86 kg.

10 a $\mathbf{R} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix}$

Since **R** is parallel to $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\mathbf{R} = \begin{pmatrix} k \\ -2k \end{pmatrix}$ where k is a constant

$\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} k \\ -2k \end{pmatrix}$

Collecting **i** terms: $2 + p = k$

Collecting **j** terms: $5 + q = -2k$

Substituting for k: $5 + q = -2\,(2 + p)$

so $5 + q = -4 - 2p$

$2p + q + 9 = 0$

b $p = 1$

From **a** above, $k = 2 + p$

so $k = 2 + 1 = 3$

so $\mathbf{R} = \begin{pmatrix} k \\ -2k \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

$|\mathbf{R}| = \sqrt{3^2 + (-6)^2} = \sqrt{45}$

$a = 15\sqrt{5}$, $F = \sqrt{45}$

$F = ma$

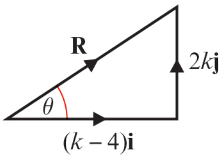
$\sqrt{45} = m \times 15\sqrt{5}$

$m = \dfrac{\sqrt{45}}{15\sqrt{5}} = \dfrac{\sqrt{9 \times 5}}{15\sqrt{5}} = \dfrac{3\sqrt{5}}{15\sqrt{5}} = \dfrac{1}{5} = 0.2$

The mass of the particle is 0.2 kg.

Challenge

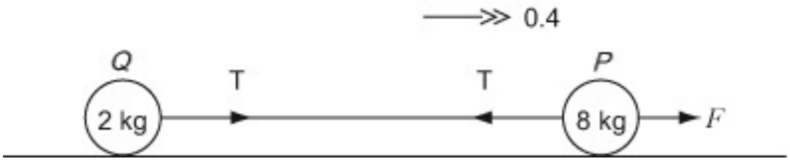
Resultant force, $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$
$\mathbf{R} = -4\mathbf{i} + (k\mathbf{i} + 2k\mathbf{j})$



$F = ma$
$m = 0.5,\ a = 8\sqrt{17}$
So magnitude of the resultant force $= 0.5 \times 8\sqrt{17} = 4\sqrt{17}$
$|\mathbf{R}|^2 = (k-4)^2 + (2k)^2$
$\left(4\sqrt{17}\right)^2 = 16 \times 17 = k^2 - 8k + 16 + 4k^2$
$272 = 5k^2 - 8k + 16$
$5k^2 - 8k - 256 = 0$
$k = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$k = \dfrac{8 \pm \sqrt{8^2 - 4 \times 5 \times (-256)}}{2 \times 5} = \dfrac{8 \pm \sqrt{5184}}{10} = \dfrac{8 \pm 72}{10}$
$k = -6.4$ or 8
Since k is given as a positive constant, the value of k is 8.

Exercise 4E

1



a $R(\rightarrow)$, $F = (2+8)\times 0.4$
$= 4$

Hence F is 4 N.

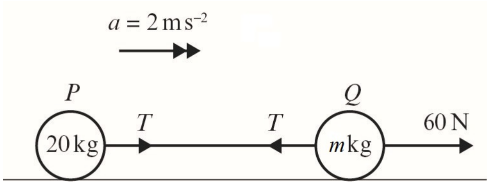
b For Q:

$R(\rightarrow)$, $T = 2\times 0.4$
$= 0.8$

The tension in the string is 0.8 N.

c Treating the string as inextensible (i.e. it does not stretch) allows us to assume that the acceleration of both masses is the same. Treating the string as light (i.e. having no/negligible mass) allows us to assume that the tension is the same throughout the length of the string and that its mass does not need to be considered when treating the system as a whole.

2

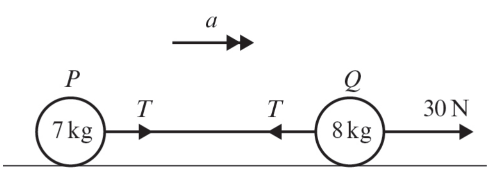


$F = ma$

a For the whole system: $F = 60$, $m = 20 + m = 10$, $a = 2$
$60 = (20 + m) \times 2$
$20 + m = 60 \div 2$
$m = 30 - 20$
The mass of Q is 10 kg.

b For P: $F = T$, $m = 20$, $a = 2$
$T = 20 \times 2$
The tension in the string is 40 N.

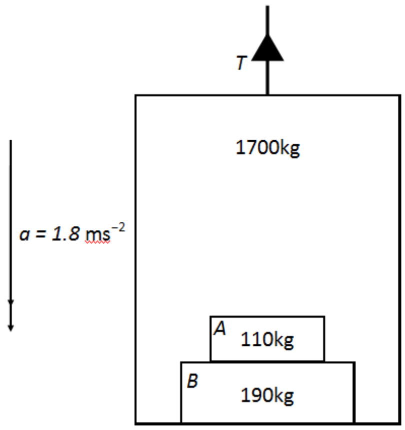
3 $F = ma$



3 **a** For the whole system: $F = 30$, $m = 8 + 7 = 15$, $a = ?$
$30 = 15a$
The acceleration of the system is 2 m s^{-2}.

b For P: $F = T$, $m = 7$, $a = 2$
$T = 7 \times 2$
The tension in the string is 14 N.

4



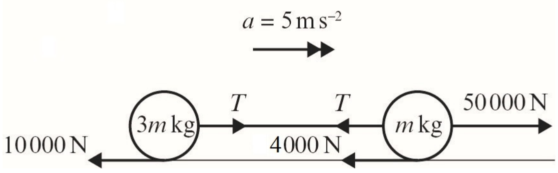
a Considering the system as a whole: total mass, m = 1700 + 110 + 190 = 2000 kg
Taking down as positive:
$F = ma = mg - T$
$2000 \times 1.8 = (2000 \times 9.8) - T$
$T = 19600 - 3600$
The tension in the cable is 16 000 N.

b **i** Force exerted on box A by box B is a normal reaction force, R_1 which acts upwards.
For box A, taking down as positive:
$110 \times 1.8 = 110g - R_1$
$R_1 = 110(g - 1.8)$
$R_1 = 110 \times 8$
Box B exerts an upwards force of 880 N on box A.

ii Let downward force exerted on lift by box B be S.
For lift alone, taking down as positive:
$1700 \times 1.8 = 1700g + S - T$
$S = T + 1700(1.8 - g)$
$S = 16\,000 - 13600 = 2400$

Alternatively (or as check), use Newton's third law of motion:
|Force exerted box B by box A| = |Force exerted on box A by box B| = 880 N
|Force exerted on lift by box B| = |Force exerted on box B by lift| = $|R_2|$
For box B, taking down as positive:
$190 \times 1.8 = 880 + 190g - R_2$
$R_2 = 880 + 190(g - 1.8)$
$R_2 = 880 + 1520 = 2400$

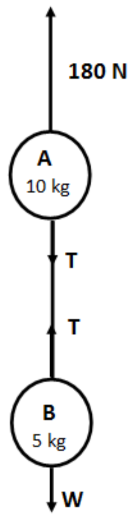
5 $F = ma$



a For the whole system:
$F = 50\,000 - 10\,000 - 4000 = 36\,000$
$a = 5$
total mass $= 3m + m = 4m$
$36\,000 = a \times$ total mass $= 4m \times 5 = 20m$
$m = 1800$
so $3m = 5400$
The mass of the lorry is 1800 kg, and that of the trailer is 5400 kg.

b For the trailer:
$F = T - 10\,000$, $m = 5400$, $a = 5$
$T - 10\,000 = 5400 \times 5 = 27\,000$
$T = 37\,000$
The tension in the tow-bar is 37 000 N.

c Treating the tow-bar as inextensible (i.e. it does not stretch) allows us to assume that the acceleration of the truck and the trailer are the same. Treating the tow-bar as light (i.e. having no/negligible mass) allows us to assume that the tension is the same throughout its length and that its mass does not need to be considered when treating the system as a whole.

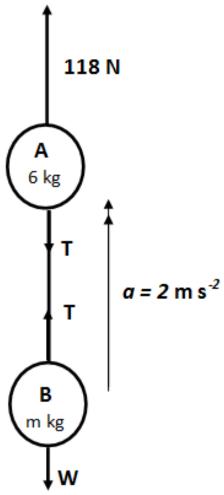


6 $F = ma$, $W = mg$
Taking upwards as positive

a For the whole system:
$180 - 15g = 15a$
$15a = 180 - (15 \times 9.8)$
$$a = \frac{180 - 147}{15} = 2.2$$
The acceleration is 2.2 m s^{-2}.

b For B:
$ma = T - W$
$5 \times 2.2 = T - (5 \times 9.8)$
$11 = T - 49$
The tension in the string is 60 N.

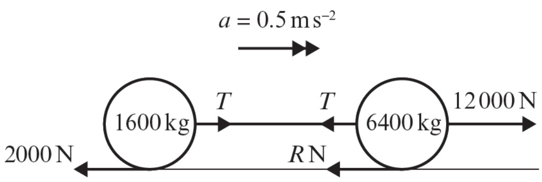
7 $F = ma,\ W = mg$
Taking up as positive



a For the whole system:
$118 - (6 + m)g = (6 + m) \times 2$
$118 = (6 + m)(2 + g) = (6 + m)(2 + 9.8)$
$\dfrac{118}{11.8} = 6 + m$
$10 = 6 + m$
The mass of B is 4 kg.

b For B:
$ma = T - W$
$4 \times 2 = T - (4 \times 9.8)$
$8 = T - 39.2$
The tension in the string is 47.2 N.

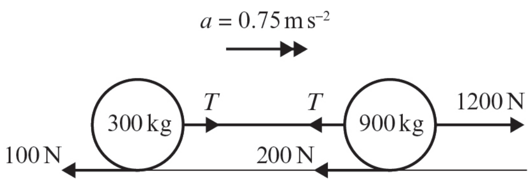
8 $F = ma$



a For the whole system:
$F = 12\,000 - 2000 - R$
$m = 1600 + 6400 = 8000$
$a = 0.5$
$10\,000 - R = 8000 \times 0.5 = 4000$
The resistance to the motion of the engine is 6000 N.

b For the carriage:
$F = T - 2000,\ m = 1600,\ a = 0.5$
$T - 2000 = 1600 \times 0.5 = 800$
The tension in the shunt is 2800 N.

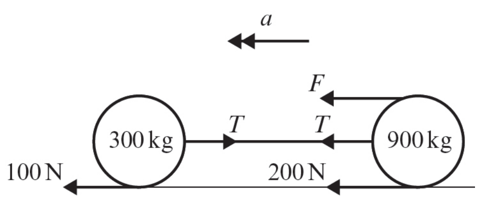
9 $F = ma$



a For the whole system:
$F = 1200 - 1000 - 200 = 900$
$m = 900 + 300 = 1200$
$900 = 1200a$
$a = 900 \div 1200 = 0.75$
The acceleration is $0.75\ \text{m s}^{-2}$, as required.

9 b For the trailer:
$F = T - 100$, $m = 300$, $a = 0.75$
$T - 100 = 300 \times 0.75 = 225$
The tension in the towbar is 325 N.

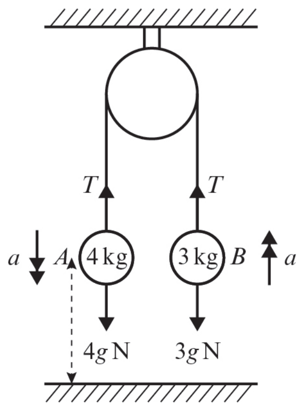
c



Taking ← as positive
Deceleration = α
Force on trailer = resistance to motion + thrust from tow-bar
Using $F = ma$
$100 + 100 = 300\alpha$
$$\alpha = \frac{200}{300} = \frac{2}{3}$$
For car:
$F + 200 - 100 = 900\alpha$
$$F = \left(900 \times \frac{2}{3}\right) - 100 = 500$$
The force the brakes produce on the car is 500 N.

Exercise 4F

1 a



For A: $R(\downarrow)$, $4g - T = 4a$ **(1)**

For B: $R(\uparrow)$, $T - 3g = 3a$ **(2)**

(1) + **(2)**: $4g - 3g = 7a$

$\Rightarrow a = \dfrac{g}{7}$

Substituting into equation (**2**):

$T = 3a + 3g = \dfrac{3g}{7} + 3g = \dfrac{24g}{7}$

$= 33.6$ N (3 s.f.)

b $u = 0$, $a = \dfrac{g}{7}$, $s = 2$, m, $v = ?$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2 \times \dfrac{g}{7} \times 2 = \dfrac{4g}{7} = 5.6$

$v = \sqrt{5.6} = 2.366...$

When A hits the ground it is travelling at 2.37 m s^{-1} (3 s.f.).

c For A: $(\downarrow)$

From part **b**, $v^2 = \dfrac{4g}{7}$

This represents the initial velocity of B when A hits the ground.

For B: $(\uparrow)$

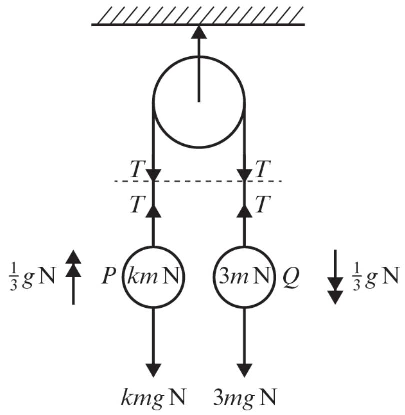
$u^2 = \dfrac{4g}{7}$, $v = 0$, $a = -g$, $s = ?$

$v^2 = u^2 + 2as$

$0 = \dfrac{4g}{7} - 2gs \Rightarrow s = \dfrac{2}{7}$

The height above the initial position is $2\frac{2}{7}$ m.

2



a For Q, $R(\downarrow)$: $3mg - T = 3m \times \frac{1}{3}g = mg$

$2mg = T$

The tension in the string is $2mg$ N.

b For P, $R(\uparrow)$: $T - kmg = km \times \frac{1}{3}g$

$3\text{T} - 3kmg = kmg$
$3\text{T} = 4kmg$
Substituting for T: $6mg = 4kmg$

$$k = \frac{6mg}{4mg}$$

The value of k is 1.5.

c Because the pulley is smooth, there is no friction, so the magnitude of acceleration of P = the magnitude of acceleration of Q.

d Up is positive.
While Q is descending, the distance travelled by $P = s_1$

$u = 0,\ a = \frac{1}{3}g,\ t = 1.8,\ s = s_1$

$s = ut + \frac{1}{2}at^2$

$s_1 = (0 \times 1.8) + \left(\frac{1}{2} \times \frac{g}{3} \times 1.8^2\right) = \frac{3.24g}{6} = 0.54g$ **(1)**

Speed of P at this time $= v_1$
Using $v^2 = u^2 + 2as$

After Q hits the ground, P travels freely under gravity and rises by a further distance s_2
$v = 0,\ u = v_1,\ a = -g,\ s = s_2$
$v^2 = u^2 + 2as$
$0^2 = 0.36g^2 - 2gs_2$

$s_2 = \frac{0.36g^2}{2g} = 0.18g$ **(2)**

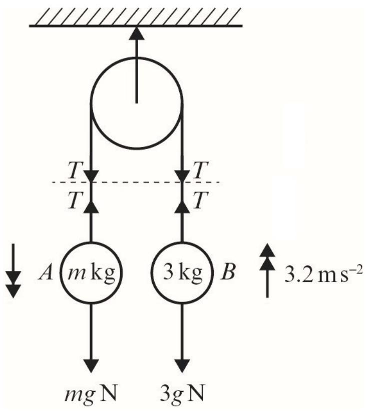
(1) + **(2)**: Total distance travelled by P from its initial position $= s_1 + s_2$

2 d P and Q are at the same height initially, so P starts at height s_1 above the plane.
Its final position = initial position + total distance travelled

$$= s_1 + (s_1 + s_2) = 2s_1 + s_2 = 2 \times 0.54g + 0.18g = 1.26g$$

P reaches a maximum height of $1.26g$ m above the plane, as required.

3



a Since the pulley is smooth, |acceleration of A| = |acceleration of B|
For A: $s = 2.5$, $u = 0$, $t = 1.25$, $a = ?$ (down is positive)

$$s = ut + \tfrac{1}{2}at^2$$

$$2.5 = (0 \times 1.25) + \tfrac{1}{2}a \times 1.25^2$$

$$a = \frac{2.5 \times 2}{1.25^2} = 3.2$$

The initial acceleration of B is 3.2 m s^{-2} as required.

b For B, $R(\uparrow)$: $T - 3g = 3a$
$T = 3(a + g) = 3(3.2 + 9.8) = 39$
The tension in the string is 39 N.

c For A, $R(\downarrow)$: $mg - T = ma$
$T = m(g - a) = m(9.8 - 3.2) = 6.6m$
Substituting for T:
$39 = 6.6m$

$$m = \frac{39}{6.6} = \frac{390}{66} = \frac{65}{11} \text{ as required}$$

d Because the string is inextensible, the tension on both sides of the pulley is the same.

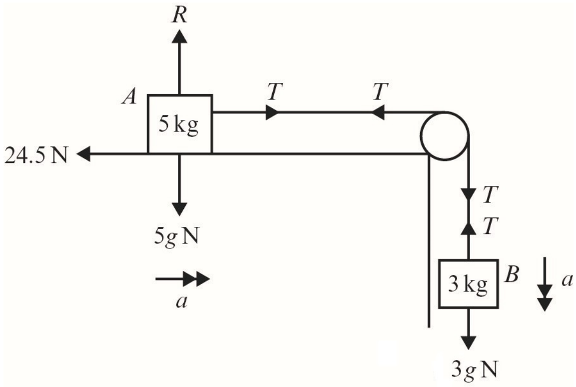
e The string will become taut again when B has risen to its maximum height and then descended to the point where A is just beginning to rise again.
If B reaches the maximum height t seconds after A hits the ground, it will also take t seconds to return to the same position as it is moving under gravity alone throughout this period. The total time of travel will be $2t$.
For B, taking up as positive, while the string is taut:
$u = 0$, $a = 1.4$, $s = 2.5$, m, $v = v_1$
$v^2 = u^2 + 2as$
$v_1^2 = 0^2 + 2 \times 3.2 \times 2.5 = 16$
Once the string is slack: $u = v_1 = 4$, $v = 0$, $a = -9.8$, $t = ?$
$v = u + at$
$0 = 4 - 9.8t$

3 e $t = \dfrac{4}{9.8} = \dfrac{40}{98} = \dfrac{20}{49}$

At this point B descends under gravity. After a further t seconds the string once again becomes taut.

The string becomes taut again $2t = \dfrac{40}{49}$ s after A hits the ground.

4



a For A: $R(\rightarrow)$, $T - 24.5 = 5a$ **(1)**

For B: $R(\downarrow)$, $3g - T = 3a$

$29.4 - T = 3a$ **(2)**

$(\mathbf{1}) + (\mathbf{2})$: $29.4 - 24.5 = 8a$

$4.9 = 8a$

$0.6125 = a$

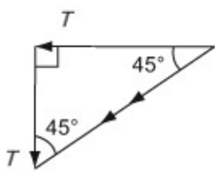
The acceleration of the system is $0.613\ \text{m s}^{-2}$ (3 s.f.)

b $T - 24.5 = 5 \times 0.6125$

$T = 27.5625$

The tension in the string is $27.6\ \text{N}$ (3 s.f.)

c



By Pythagoras,

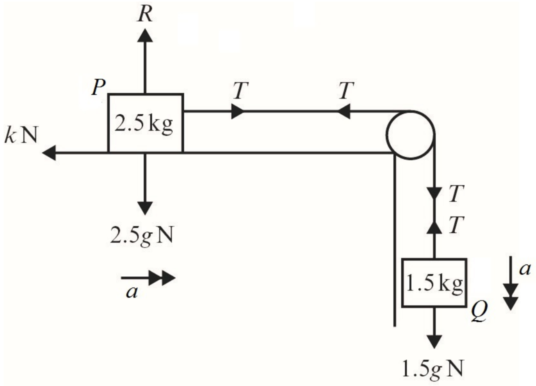
$F^2 = T^2 + T^2 = 2T^2$

$F = T\sqrt{2} = 27.5625 \times \sqrt{2}$

$= 38.979\ldots$

The magnitude of the force exerted on the pulley is $39\ \text{N}$ (2 s.f.)

5



a **i** For Q: $s = 0.8$, $u = 0$, $t = 0.75$, $a = ?$ (down is positive)

$s = ut + \frac{1}{2}at^2$

$0.8 = (0 \times 0.75) + \frac{1}{2}a \times 0.75^2$

$a = \frac{0.8 \times 2}{0.75^2} = 2.844...$

The acceleration of Q is 2.84 m s^{-2} (3 s.f.)

ii For Q, $R(\downarrow)$: $1.5g - T = 1.5a$

$T = 1.5(g - a) = 1.5(9.8 - 2.84) = 10.44$

The tension in the string is 10.4 N (to 3 s.f.), as required.

iii For P, $R\ (\rightarrow)$: $T - k = 2.5a$

Substituting:

$10.4 - k = 2.5 \times 2.84$

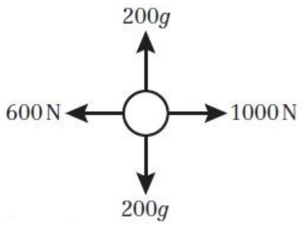
$k = 10.4 - 7.1$

The value of k is 3.3 N

b Because the string is inextensible, the tension in all parts of it is the same.

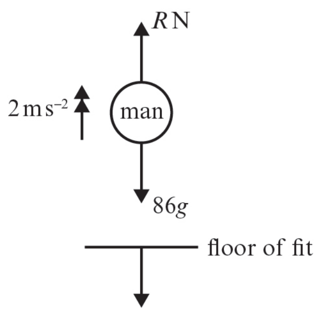
Chapter review 4

1 a



b Vertical forces can be ignored as they are in equilibrium and at right angles to the direction of interest.
$F = ma$
$m = 200$, Resultant force, $F = 1000 – 200 – 400 = 400$
$400 = 200a$
The acceleration of the motorcycle is 2 m s^{-2}.

2



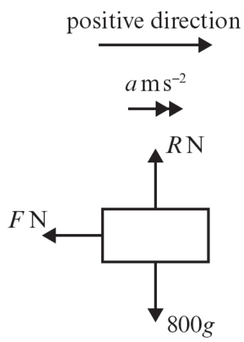
For the man

$$R(\uparrow),\quad R - 86g = 86\times 2$$
$$R = 86\times 9.8 + 86\times 2$$
$$= 1014.8 \approx 1000$$

The reaction on the man on the floor is of equal magnitude to the action of the floor on the man and in the opposite direction.

The force that the man exerts on the floor of the lift is of magnitude 1000 N (2 s.f.) and acts vertically downwards.

3



3 **a** $u = 18,\ v = 12,\ t = 2.4,\ a = ?$

$$v = u + at$$
$$12 = 18 + 2.4a$$
$$a = \frac{12-18}{2.4} = -2.5$$

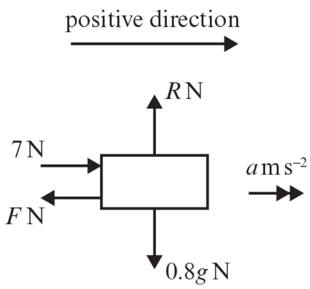
$$F = ma$$
$$-F = 800 \times -2.5 = -2000$$
$$F = 2000 \text{ N}$$

b $u = 18,\ v = 12,\ t = 2.4,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t$$
$$= \left(\frac{18+12}{2}\right) \times 2.4$$
$$= 15 \times 2.4 = 36$$

The distance moved by the car is 36 m

4



a $u = 2,\ v = 4,\ s = 4.8,\ a = ?$

$$v^2 = u^2 + 2as$$
$$4^2 = 2^2 + 9.6a$$
$$a = \frac{16-4}{9.6} = 1.25$$

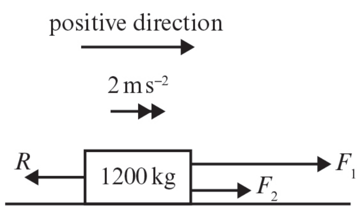
The magnitude of the acceleration of the block is 1.25 m s^{-2}

b $R(\uparrow),\quad F = ma = 0.8 \times 1.25 = 1$

$R(\rightarrow),\quad 7 - F = 6$

The magnitude of the frictional force between the block and the floor is 6 N.

5



5 Let R = the resistive force
Let F_1 = the driving force
Let F_2 = the resultant force

$F_2 = ma = 1200 \times 2 = 2400$

$F_1 = 3R \Rightarrow R = \frac{1}{3}F_1$

The driving force is the resultant force plus the resistive force:

$F_1 = R + F_2 = \frac{1}{3}F_1 + 2400$

$\frac{2}{3}F_1 = 2400$

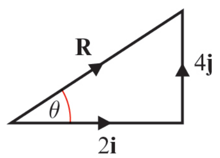
$F_1 = 3600$

The magnitude of the driving force is 3600 N, as required.

6 $\mathbf{F}_1 = (3\mathbf{i} + 2\mathbf{j})$, $\mathbf{F}_2 = (4\mathbf{i} - \mathbf{j})$, $m = 0.25$
$F = \mathbf{F}_1 + \mathbf{F}_2 = ma$
$(3\mathbf{i} + 2\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) = 0.25a$
$(7\mathbf{i} + \mathbf{j}) = 0.25a$
$a = \frac{(7\mathbf{i} + \mathbf{j})}{0.25}$
The acceleration is $(28\mathbf{i} + 4\mathbf{j})$ m s^{-2}.

7 $\mathbf{F}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\mathbf{F}_2 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $\mathbf{F}_3 = \begin{pmatrix} a \\ -2b \end{pmatrix}$ $m = 2,\ a = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$
$F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = ma$
$\begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} a \\ -2b \end{pmatrix} = 2\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$
Considering **i** components: $2 + 3 + a = 6$
$a = 6 - 5$
Considering **j** components: $-1 - 1 - 2b = 4$
$-2b = 4 + 2$
The values of a and b are 1 and −3, respectively.

8



a $|R| = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$
Using $F = ma$
$2\sqrt{5} = 2a$
The acceleration of the sled is $\sqrt{5}$ m s^{-2}.

b $u = 0$, $t = 3$, $a = \sqrt{5}$, $s = ?$
$s = ut + \frac{1}{2}at^2$
$s = (0 \times 3) + \left(\frac{1}{2} \times \sqrt{5} \times 3^2\right) = \frac{9\sqrt{5}}{2}$

8 b The sled travels a distance of $\frac{9\sqrt{5}}{2}$ m.

9 a Since object is in equilibrium, $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

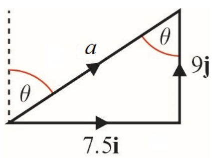
$(3a\mathbf{i} + 4b\mathbf{j}) + (5b\mathbf{i} + 2a\mathbf{j}) + (-15\mathbf{i} - 18\mathbf{j}) = 0$
Collecting **i** terms: $3a + 5b = 15$ **(1)**
Collecting **j** terms: $2a + 4b = 18$ **(2)**
Subtracting (**2**) from (**1**) gives $a + b = -3$
Therefore $b = -3 - a$

Substituting this into (**1**):
$3a + 5(-3 - a) = 15$
$3a - 15 - 5a = 15$
$-2a = 30$
$a = -15$
Substituting this into (**1**):
$3(-15) + 5b = 15$
$5b = 15 + 45 = 60$
$b = 12$

The values of a and b are -15 and 12, respectively.

b i $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$, so when $\mathbf{F}_3$ is removed, the resultant force $F = -\mathbf{F}_3$
i.e. $F = (15\mathbf{i} + 18\mathbf{j})$

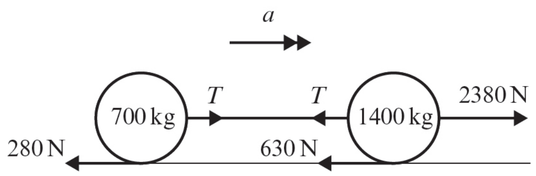
$m = 2$
$F = ma$
$(15\mathbf{i} + 18\mathbf{j}) = 2a$
$a = (7.5\mathbf{i} + 9\mathbf{j})$



$|a| = \sqrt{7.5^2 + 9^2} = \sqrt{137.25}$
Using Z angles (see diagram), bearing = θ
$\tan\theta = \frac{7.5}{9}$
The magnitude of the acceleration is 11.7 m s^{-2} and it has a bearing of 039.8° (both to 3 s.f.).

ii $u = 0$, $t = 3$, $a = 11.7$, $s = ?$
$s = ut + \frac{1}{2}at^2$
$s = (0 \times 3) + \left(\tfrac{1}{2} \times 11.7 \times 3^2\right) = \frac{105.3}{2}$
The object travels a distance of 52.7 m (to 3 s.f.).

10



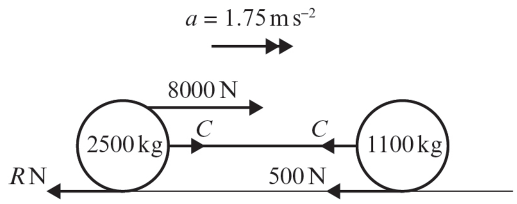
a $F = ma$
For the whole system:
$F = 2380 - 630 - 280 = 1470$
$m = 1400 + 700 = 2100$
$1470 = 2100a$
Since the tow-rope is inextensible, the acceleration of each part of the system is identical.
The acceleration of the car is 0.7 m s^{-2}.

b For the trailer:
$F = T - 280$, $m = 700$, $a = 0.7$
$T - 280 = 700 \times 0.7 = 490$
The tension in the tow-rope is 770 N.

c For the car, after the rope breaks:
resultant force = 2380 – 630 = 1750
$m = 1400$
therefore $a = 1750 \div 1400 = 1.25$
$u = 12$
$s = ut + \frac{1}{2}at^2$
$s = (12 \times 4) + \left(\frac{1}{2} \times 1.25 \times 4^2\right) = 48 + 10$
In the first 4 s after the tow-rope breaks, the car travels 58 m.

d Since the tow-rope is inextensible, the tension is constant throughout the length, and the acceleration of each part of the system is identical.

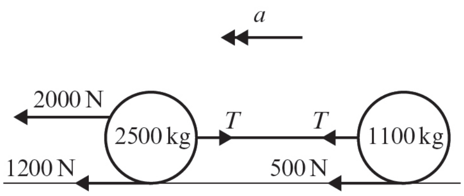
11



a $F = ma$
For the whole system:
$F = 8000 - 500 - R = 7500 - R$
$m = 2500 + 1100 = 3600$
$a = 1.75$
$7500 - R = 3600 \times 1.75 = 6300$
$R = 7500 - 6300$
The resistance to the motion of the train is 1200 N, as required.

b Considering the carriage only:
$C - 500 = 1100 \times 1.75 = 1925$
The compression force in the shunt is 2425 N.

11 c



Taking ← as positive
Deceleration = α
Force on carriage = resistance to motion + thrust in shunt
Using $F = ma$
$500 + C = 1100\alpha$

$$\alpha = \frac{500 + C}{1100}$$

For engine:
$2000 + 1200 - C = 2500\alpha$
Substituting for α:

$$3200 - C = 2500 \times \left(\frac{500 + C}{1100}\right)$$

$$11\cancel{00}(3200 - C) = 25\cancel{00}(500 + C)$$

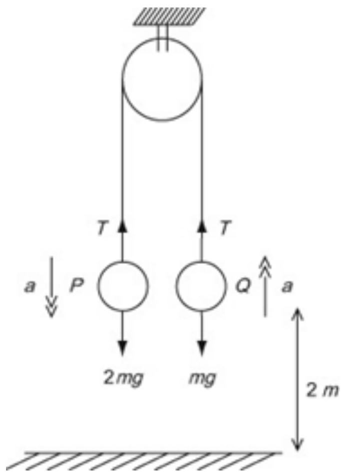
$$35200 - 11C = 12500 + 25C$$

$$35200 - 12500 = 11C + 25C$$

$$C = \frac{22700}{36}$$

The thrust in the shunt is 630 N (2 s.f.).

12 a



For P: $R(\downarrow)$, $\quad 2mg - T = 2ma$
For Q: $R(\uparrow)$, $\quad T - mg = ma$
Add, $\quad mg = 3ma$

$$a = \tfrac{1}{3} g \text{ m s}^{-1}$$

12 b For P:

$$v^2 = u^2 + 2as$$
$$v^2 = 0 + 2 \times \tfrac{1}{3} g \times 2$$
$$v = \sqrt{\frac{4g}{3}}$$
$$= 3.6\,\text{m s}^{-1} \quad (2\text{ s.f.})$$

c For Q:

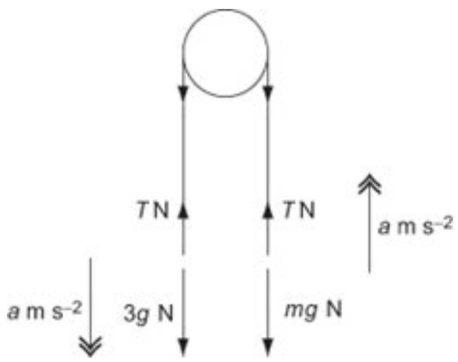
$$R(\uparrow), \quad -mg = ma$$
$$a = -g$$
$$v^2 = u^2 + 2as \quad (\uparrow),$$
$$0 = \frac{4g}{3} - 2gs$$
$$s = \tfrac{2}{3}\,\text{m}$$

$\therefore$ Height above the ground $= 2\tfrac{2}{3}$ m

d i In an extensible string $\Rightarrow$ acceleration of both masses is equal.

ii Smooth pulley $\Rightarrow$ same tension in string either side of the pulley.

13 a



For the 3 kg mass

$$R(\downarrow), \quad F = ma$$
$$3g - T = 3 \times \tfrac{3}{7} g$$
$$T = 3g - \tfrac{9}{7} g = \tfrac{12}{7} g$$

The tension in the string is $\tfrac{12}{7} g$ N

13 b For the m kg mass

$R(\uparrow)$, $\quad F = ma$

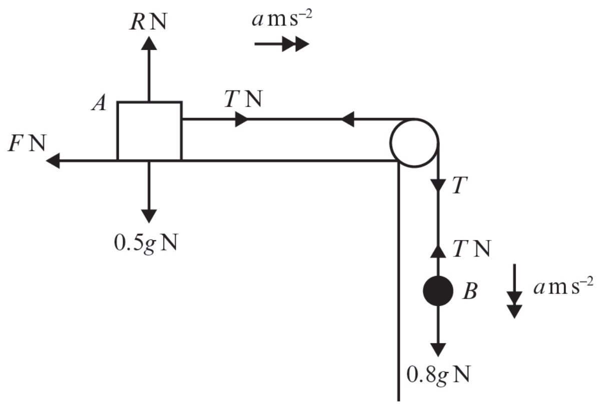
$$T - mg = m \times \tfrac{3}{7}g$$

Using the answer to **a**

$\tfrac{12}{7}g - mg = \tfrac{3}{7}mg$

$\tfrac{12}{7} = \tfrac{10}{7}m \Rightarrow m = 1.2$

14



a For B:

$u = 0,\ s = 0.4,\ t = 0.5,\ a = ?$

$s = ut + \tfrac{1}{2}at^2$

$0.4 = 0 + \tfrac{1}{2}a \times 0.5^2 = \tfrac{1}{8}a$

$a = 8 \times 0.4 = 3.2$

The acceleration of B is $3.2\,\text{m s}^{-2}$

b For B:

force $= ma$

$0.8g - T = 0.8 \times 3.2$

$T = 0.8 \times 9.8 - 0.8 \times 3.2$

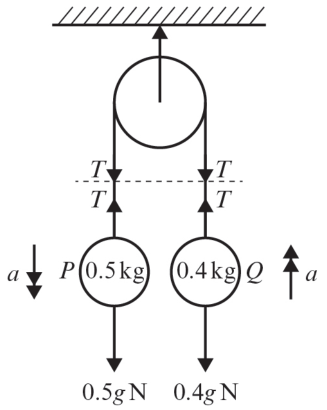
$= 5.28$

The tension in the string is 5.28 N (2 s.f.). (As the numerical value $g = 9.8$ has been used, you should correct your answer to 2 significant figures.)

c $F = 3.7$ (2 s.f.)

d The information that the string is inextensible has been used in part **c** when the acceleration of A has been taken to be equal to the acceleration of B.

15



a **i** For P, $R(\downarrow)$: $0.5g - T = 0.5a$ **(1)**

ii For Q, $R(\uparrow)$: $T - 0.4g = 0.4a$ **(2)**

b (**1**) × 4: $2g - 4T = 2a$
(**2**) × 5: $5T - 2g = 2a$
Equating these:
$2g - 4T = 5T - 2g$
$9T = 4g$
The tension in the string is $\frac{4}{9}g$ N (4.35 N).

c Using equation (**1**):
$\frac{1}{2}g - \frac{4}{9}g = \frac{1}{2}a$
$g - \frac{8}{9}g = a$
The acceleration is $\frac{1}{9}g$ m s^{-2} (1.09 m s^{-2} (3 s.f.)).

d When the string breaks, Q has moved up a distance s_1 and reached a speed v_1
Now Q moves under gravity (after the string breaks) initially upwards.
To reach the floor it has to travel a distance $s = 2 + s_1$

While the string is intact, up positive:

$u = 0,\ t = 0.2,\ a = \dfrac{g}{9},\ s_1 = ?$

$$s_1 = ut + \frac{1}{2}at^2$$
$$= (0 \times 0.2) + \left(\frac{1}{2} \times \frac{g}{9} \times 0.2^2\right)$$
$$= \frac{g}{450}$$

$$v_1 = u + at$$
$$= 0 + \frac{g}{9} \times 0.2$$
$$= \frac{g}{45}$$

15 d So, when the string breaks, Q is $2+\dfrac{g}{450}$ above the ground, a moving upwards with a speed of $\dfrac{g}{45}$.

After string breaks, Q moves under gravity. So taking down as positive, for the motion after the string breaks, we have

$$u = v_1 = -\frac{g}{45},\ a = g,\ s = 2+\frac{g}{450},\ t = ?$$

$$s = ut + \tfrac{1}{2}at^2$$

$$2+\frac{g}{450} = -\frac{g}{45}t + \frac{1}{2}gt^2$$

$$\frac{(900+g)}{450} = -\frac{g}{45}t + \frac{1}{2}gt^2$$

$$0 = \frac{1}{2}gt^2 - \frac{g}{45}t - \frac{(900+g)}{450}$$

Let $g = 9.8 \Rightarrow 4.9t^2 - 0.21\dot{7}\dot{8}t - 2.02178 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-0.2178 \pm \sqrt{(-0.2178)^2 - (4\times 4.9\times -2.02178)}}{2\times 4.9}$$

$$= \frac{-0.218 \pm \sqrt{39.674}}{9.8}$$

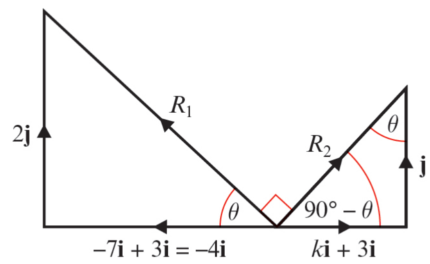
$$= 0.66\text{ s or } -0.621\text{ s}$$

Only the positive root is relevant: $t = 0.66$ (2 s.f.)

Q hits the floor 0.66 s after the string breaks.

Challenge

Total force on first boat: $R_1 = (-7\mathbf{i} + 2\mathbf{j}) + 3\mathbf{i} = -4\mathbf{i} + 2\mathbf{j}$
Total force on second boat: $R_2 = (k\mathbf{i} + \mathbf{j}) + 3\mathbf{i} = (k + 3)\mathbf{i} + \mathbf{j}$
Since mass is a vector quantity, the acceleration of each boat will be parallel to the resultant force acting on it, so the relationship between the components of the accelerations is as shown in the diagram below.



From R_1: $\tan\theta = \frac{2}{4} = \frac{1}{2}$

From R_2: $\tan\theta = \frac{k+3}{1} = k+3$

Equating these: $\frac{1}{2} = k+3$

$2k + 6 = 1$

$2k = -5$

The value of k is -2.5.